TEACHING SEQUENCE ABOUT TESSELLATION
Première - Terminale

I.R.E.M. de Clermont-Ferrand
Groupe Maths en Anglais

2015 - 2016

Ont collaboré à l'élaboration de cette séquence :

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Préambule

Le cours de Mathématiques en Anglais, dispensé dans le cadre de la DNL, est l'occasion d'aborder des thèmes absents des programmes.

Ce livret présente un ensemble d'activités autour des pavages, réunies dans une même séquence s'adressant à des élèves de Première ou Terminale, et aboutissant à la preuve de l'existence de trois pavages réguliers du plan (et trois seulement).

Les activités proposées donnent un panel de travaux envisageables en classe de DNL, autour des compétences linguistiques suivantes : compréhension orale, compréhension écrite, expression orale, expression écrite.

Elles ont toutes été testées en classe et modifiées après essai.

Les objectifs et le scénario de ces activités sont détaillés, en anglais, dans la version professeur. En annexe, figure la version élèves.

Nous remercions chaleureusement Susan Leahy pour son travail de relecture.

TESSELLATION

"For me it remains an open question whether [this work] pertains to the realm of mathematics or to that of art."

M.C. Escher

Teaching Sequence about Tessellation

Table of Contents

Teaching Sequence about Tessellation **Overall Organization**	1ère ou Term euro 5 to 6 hours

Keywords: Tiling; tessellation; regular polygons.

Outline: This sequence is dedicated to the problem of tessellation, using regular polygons to tile the plane, without any gaps or overlap.

Mathematical skills: Organising – Conjecturing – Reasoning – Justifying.

Language skills: Reading - Listening – Writing - Speaking.

Prerequisites: The basic vocabulary of plane geometry.

Materials: Overhead projector. Copies of a projector.

Contents: The sequence consists of 4 dependent activities lasting 5 to 6 hours.

Activity 1: Guessing the lesson. **1 hour**

Starting from a set of clues presented on a word cloud, learners have to guess what the lesson is going to be about. Afterwards, a short video completes the discussion.

Activity 2: Focus on language. **1 hour 30 min**

Using a text that defines the specific vocabulary of the lesson, learners have to match illustrations to expressions. As a conclusion, the keywords of the lesson are gathered in a crossword.

Activity 3: Writing frame. **1 hour**

Learners work in pairs. They conjecture the different types of regular tilings, using regular polygons cut out of cardboard. They have to recap their results on a poster, and present it to another group.

Activity 4: A proof. **1 hour 30 min**

We focus on the assumption that only the regular tilings consist of equilateral triangles, squares and hexagons. Learners have to complete the different steps of the proof and put them in order. We insist on the connection words and the expressions used in mathematical proofs.

Teaching Sequence about Tessellation **Activity 1: Guessing the lesson**	1ère ou Term euro 1 hour

Keywords: tessellation; tiling; polygons.

Outline: Learners guess and discuss what the lesson is going to be about from a set of clues presented in a word cloud. Afterwards, a video permits them to complete their answers.

Mathematical skills: Guessing - Hypothesizing.

Language focus: Nouns or questions related to the topic students are about to learn.

Language skills: Listening – Speaking.

Prerequisites: Basic vocabulary of plane geometry.

Materials: To each learner a word cloud with questions (doc 1) and a video questions list (doc 2); overhead projector; video file.

Preparation and procedure:

Part 1: Word cloud **30 min**

- ✓ Learners receive a word cloud with some keywords and phrases related to the topic.
- ✓ Learners look at the words and answer questions such as the following:
  - What do you think the lesson will be about?
  - Which words do you know?
  - Which words can you add to these?
- ✓ Learners discuss the answers together.

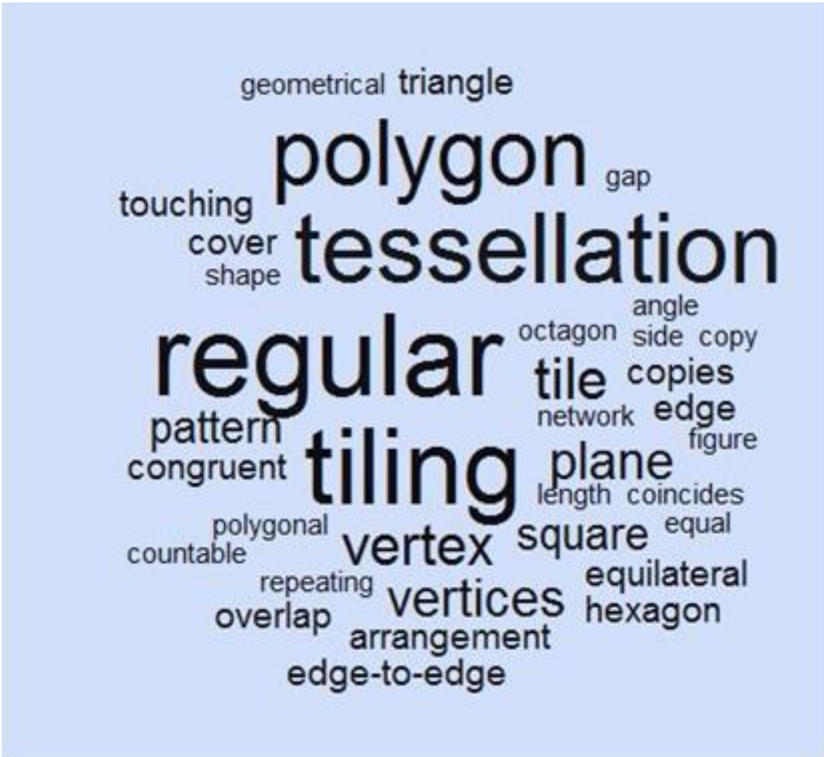
Part 2: Video **30 min**

The Mathematical Art of M.C. Escher (de 0.00 à 1.13)
Ian Stewart University of Warwick
BBC 4 (2005)

Video link http://www.youtube.com/watch?v=Kcc56fRtrKU

- ✓ Learners watch the video and discuss what they understood.
- ✓ Learners receive a document with some questions, watch the video one more time and answer the questions.
- ✓ We discuss together the video in general.

Teaching Sequence about Tessellation **Activity 1: Guessing the lesson**	
Doc. 1	**Word Cloud**



1) What do you think the lesson will be about?
2) Which words do you know?
3) Which words can you add to these?

Teaching Sequence about Tessellation **Activity 1: Guessing the lesson**	
Doc. 2	**Video Questions**

The Mathematical Art of M.C. Escher

Ian Stewart, University of Warwick

BBC 4 (2005)

1) Who was Escher?

2) Where did Escher find inspiration?

3) What is a tessellation?

4) Why is tessellation about Mathematics?

Teaching Sequence about Tessellation

<u>Activity 1</u>: Correction

Part 1: Word Cloud

The vocabulary required

Polygon: a plane figure bounded by line segments. Some polygons: triangle (3), quadrilateral (4), pentagon (5), hexagon (6), heptagon (7), octagon (8), enneagon (9), decagon (10), undecagon (11), dodecagon (12).

Regular polygon: a polygon with all sides and all angles equals.

Edge: a line along which two faces or surfaces of a solid meet (edge-to-edge = *bord à bord*).

Congruent: having identical shapes so that all parts correspond, corresponding exactly when superimposed.

Vertex: The point at which the sides of an angle intersect.

Shape: the contour of a thing.

Tile: a piece of baked clay used to form a roof, to cover a wall.

Tiling: something made with tiles, a tiled surface (*pavage, carrelage*).

Tessellation: a kind of tiling.

Pattern: a decorative design made up of elements in a regular arrangement (*motif*).

Arrangement: the act of arranging or being arranged.

To arrange: to place in proper order.

Gap: a space between objects or points.

Overlap: to cover part of, or something.

Teaching Sequence about Tessellation

Activity 1: Correction

Part 2: Video

Transcript

The Mathematical Art of M.C. Escher

Ian Stewart, University of Warwick

BBC 4 (2005)

"An amazing thing about MC Escher is that he represents the perfect coming together of mathematics and arts. These are two different worlds but in his work they are brought together as one.

Born in the Netherlands in 1898, Mauritius Cornelius Escher had no formal training in mathematics. He began his professional life as a graphic artist: making woodcuts and lithographs. As a young man while visiting the Alhambra in Spain, he became fascinated by the geometric decoration of the Moorish tiles. It would be a defining moment for Escher as an artist. From then on, he would spend much of his life experimenting with the area of mathematics known as tessellation.

Tessellation is about regular patterns that divide the plane. That means they fit if they split the plane up into lot of different tiles and those tiles fit together perfectly: they don't overlap and they don't leave any gaps. It may seem that the premise of tiling a plane or surface with regular repeating unit is a very simple idea. But it's absolutely fundamental to mathematics and the reason is that it's about symmetry. "

Questions

1) Who was Escher?

Escher was a Dutch artist of the 20th century. He made woodcuts and lithographs. He is considered as the perfect coming together of mathematics and arts. In his work he brought the two worlds together as one, experimenting on tessellation.

2) Where did Escher find inspiration?

He discovered geometric decoration of the Moorish tiles in the Alhambra in Spain. From then on he spent much of his life experimenting with tessellation.

3) What is a tessellation?

Tessellation is about regular patterns that divide the plane. They fit if they do not over-lap and if they don't leave any gaps.

4) Why is tessellation about mathematics?

Even if it may seem that tiling a plane with a regular repeating pattern is a very simple idea, it's fundamental because it's about symmetry.

Vocabulary

Coming together*: the act of joining together as one*

To bring together*: to join*

Woodcuts*: a block of wood cut along the grain and with a design (gravure)*

Lithographs*: a print by lithography*

Premise*: preliminary fact*

Teaching Sequence about Tessellation **Activity 2: Focus on language**	1ère ou Term euro 1 hour 30 min

Keywords: tiling; tessellation; regular polygons; congruent.

Outline: Starting from a text explaining the vocabulary, learners match words (or expressions) to the appropriate illustrations. They have to explain their choice. As a recap they fill in a crossword with the keywords of the lesson.

Mathematical skills: Understanding a definition – Reasoning - Justifying.

Language focus: Specific vocabulary of the lesson.

Language skills: Reading – Understanding – Speaking – Giving opinions.

Prerequisites: Vocabulary seen previously in activity 1.

Materials: For each learner a text and two tables: one with illustrations, the other to fill in (doc 1), a crossword (doc 2).

Preparation and procedure:

Part 1: Text and matching exercise **1 hour**

- ✓ Learners read the text carefully.
- ✓ Learners have to match expressions from the text to illustrations, and explain their choice by quoting from the text.
- ✓ As a conclusion, you can select some illustrations on the Internet and show them to the learners so that they recognize the kind of tiling they represent.

Part 2: Crossword **30 min**

- ✓ Learners read the definition and complete the crossword using words and expressions they came across before.

Teaching Sequence about Tessellation **Activity 2: Focus on language**	
Doc. 1	**Text and matching**

a) Read the text "Plane tiling and tessellations" carefully.

b) Some expressions, explained in the text, are illustrated in table n°1.

Fill in table n°2 by associating the appropriate illustration to the expression. You will have to justify your choice by quoting the text.

Plane tiling and tessellations

Tiling of the plane is any countable family of tiles that covers the plane without gaps or overlaps. In this section, our tiles will be polygons.
An edge-to-edge tiling is a polygonal tiling in which each edge of each polygon coincides with an edge of exactly one other polygon, with the vertices of one tile touching only the vertices of others.
An edge-to-edge tiling in which all the tiles are regular polygons (that is to say a shape having sides of the same length and equal angles), is called a tessellation.
A regular tiling is a tessellation in which all the tiles are congruent, and with the same arrangement of polygons at each vertex.
When two or more types of regular polygons are used, with the same pattern at each vertex (the regular polygons should always appear in the same order), we talk about semi-regular tiling.

(Adapted from the book: Charming Proof - A Journey Into Elegant Mathematics, by Claudi Alsina and Roger B. Nelsen - MAA)

Table n°1

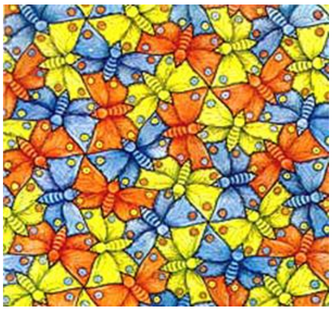
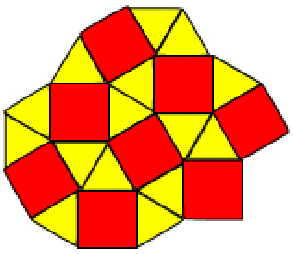
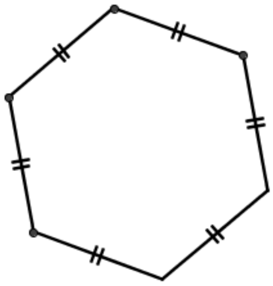
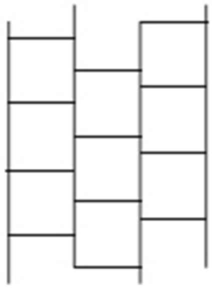
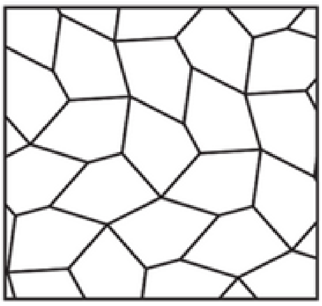
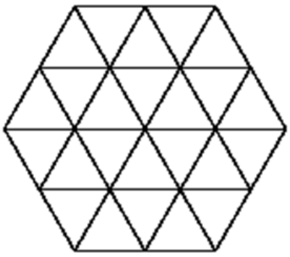
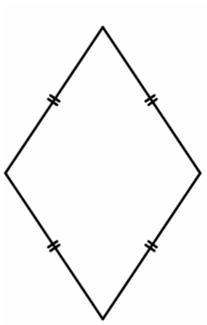
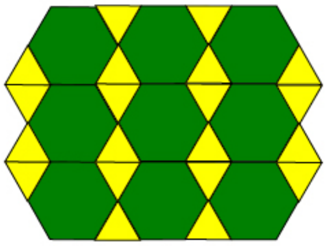
A. 	B. 	C. 	D. 
E. 	F. 	G. 	H. 

Table n°2

Expression from the text	**Illustration**	**Justify your choice by quoting the text**
A non-regular polygon.		
A regular polygon.		
A tiling which is not a polygonal one.		
A polygonal tiling which is not an edge-to-edge one.		
An edge-to-edge polygonal tiling which is not a tessellation.		
A tessellation that is not a regular tiling.		
A regular tiling.		
A tessellation that is not a semi-regular tiling.		
A semi-regular tiling.		

Teaching Sequence about Tessellation

Activity 2: Focus on language

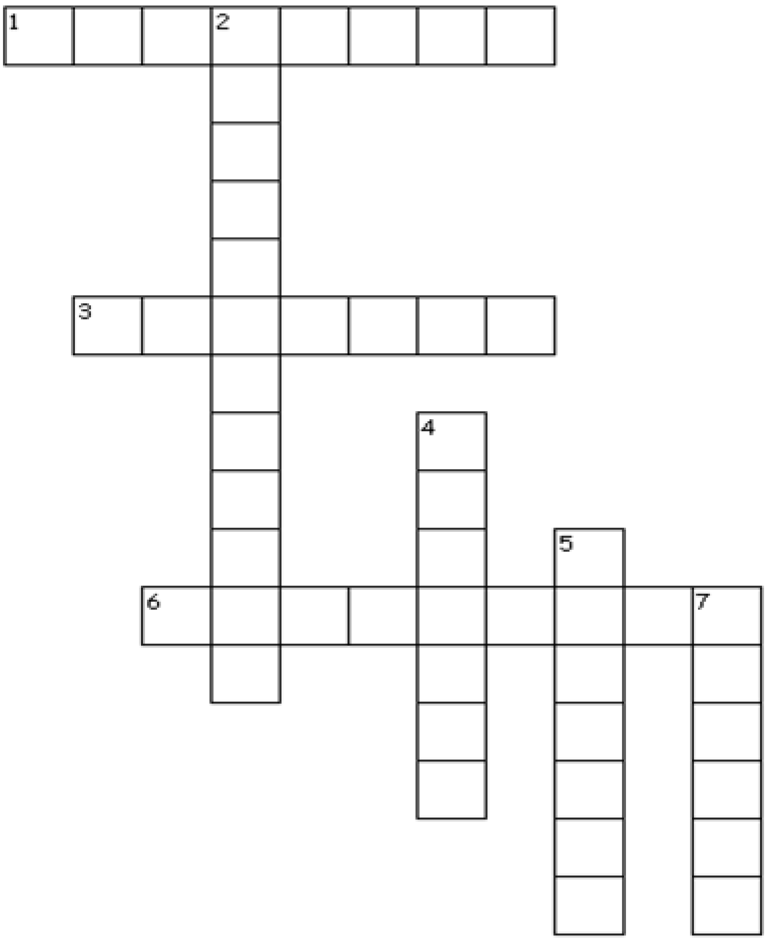
Doc. 2	Crossword

Across

1. The points where lines meet to form an angle.
3. Any 2D shape with straight sides.
6. Describes a shape in mathematics that has the same shape and size as another.

Down

2. Pattern resulting from the arrangement of regular polygons to cover a plane without any gaps.
4. To cover something partly by going over its edge; to cross each other.
5. Squares and equilateral triangles are, but not rectangles.
7. … was a favorite means of expression for the Dutch artist M. C. Escher (1898-1972).



http://puzzlemaker.discoveryeducation.com/CrissCrossSetupForm.asp

Teaching Sequence about Tessellation **Activity 3: Writing frame**	$1^{ère}$ ou Term euro 1 hour

Keywords: Regular tiling.

Outline: Learners work in pairs. They have to complete a writing frame and present it on a poster.

Mathematical skills: Reasoning – Organising – Conjecturing – Justifying.

Language focus: Giving opinions – Agreeing.

Language skills: Writing – Speaking.

Prerequisites: The previous activities.

Materials: To each learner a writing frame (doc 1). To each group some sets of regular polygons cut out of a cardboard (doc 2); a poster and pens.

Preparation and procedure:

- ✓ Prepare a writing frame, and make enough copies for everyone in the class, and a poster (one for two).
- ✓ Prepare regular polygons (from 3 to 8 sides), cut out of cardboard and make enough for each pair of learners.
- ✓ Give each learner a copy of the writing frame, and a set of regular polygons for each pair.
- ✓ Learners work in pairs and discuss how they will complete the writing frame. They agree in the group to sum up their answers on the poster.
- ✓ Ask one or two pairs to present their work to the others, or gather two pairs in order they present their poster each other.
- ✓ Discuss as a class the idea of the proof in order to prepare Activity 4.

Teaching Sequence about Tessellation **<u>Activity 3</u>: Writing frame**	
<u>Doc.1</u>	**Report describing the different types of regular tiling.**

a) Work in pairs.
b) Agree with the definition of a regular tiling.
c) Use the different sets of regular polygons to find out the different types of regular tiling, and an example of a regular polygon which can not give a regular tiling (try to explain why it does not work).
d) Sum up your results on a poster, following the guideline below. You will have to present it to another group.

Our names: 1) …………………………………………

2) ………………………………………….

Definition of a regular tiling:

………
………

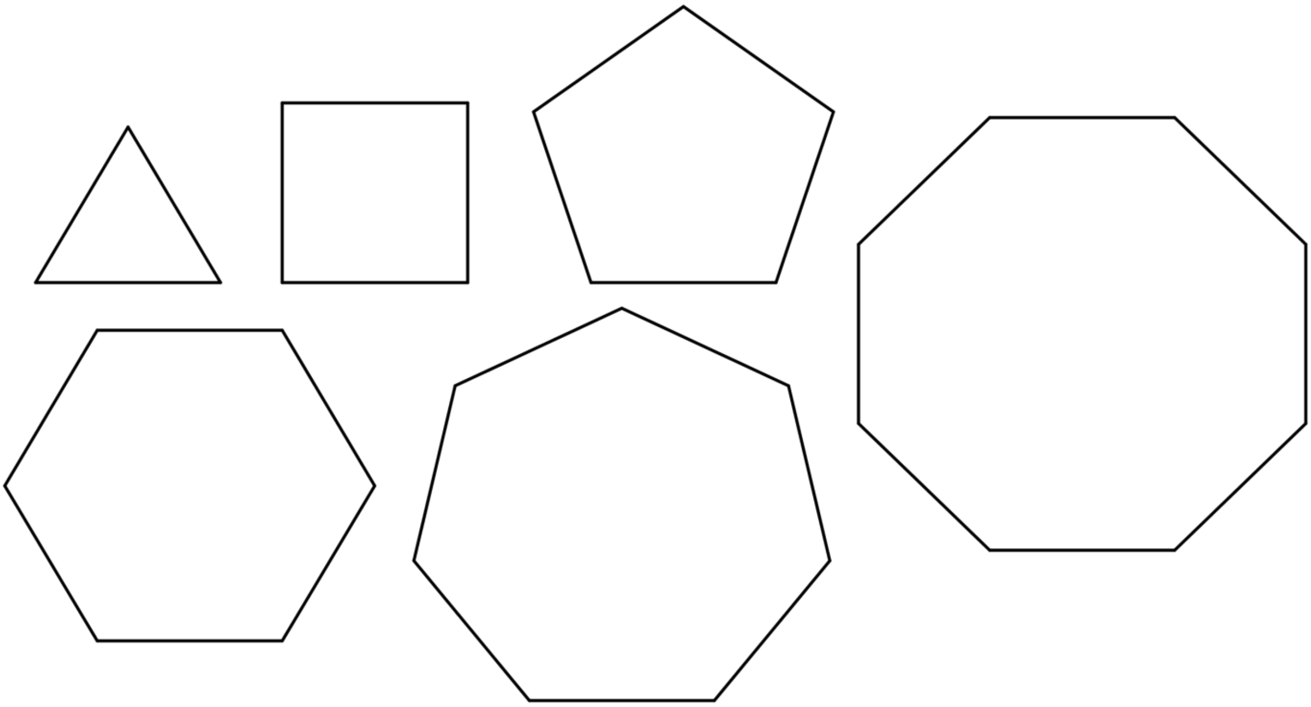
Different types of regular tiling:

Illustrations	Polygons involved

Example of a regular polygon which can not give a regular tiling.

Illustrations	Type of polygon involved	Reason why it does not work

Teaching Sequence about Tessellation **<u>Activity 3</u>: Writing frame**	
<u>Doc.2</u>	**A set of regular polygons.**



Teaching Sequence about Tessellation **Activity 4: A proof**	1ère ou Term euro 1 hour 30 min

Keywords: regular tiling; regular polygons.

Outline: Learners work in pairs. They have to complete a proof and put in order the different steps.

Mathematical skills: Understanding – Reasoning - Organising.

Language focus: Expressions and connection words used in a mathematical proof: since, then, therefore, let… be…, if and only if, assuming that….

Language skills: Reading – Understanding – Speaking.

Prerequisites: Regular tiling; regular polygons.

Materials: To each pair of learners, an introduction to the problem and the instructions (doc 1), a set of cards containing the different steps of the proof (doc 2), a correction (doc 3).

Preparation and procedure:

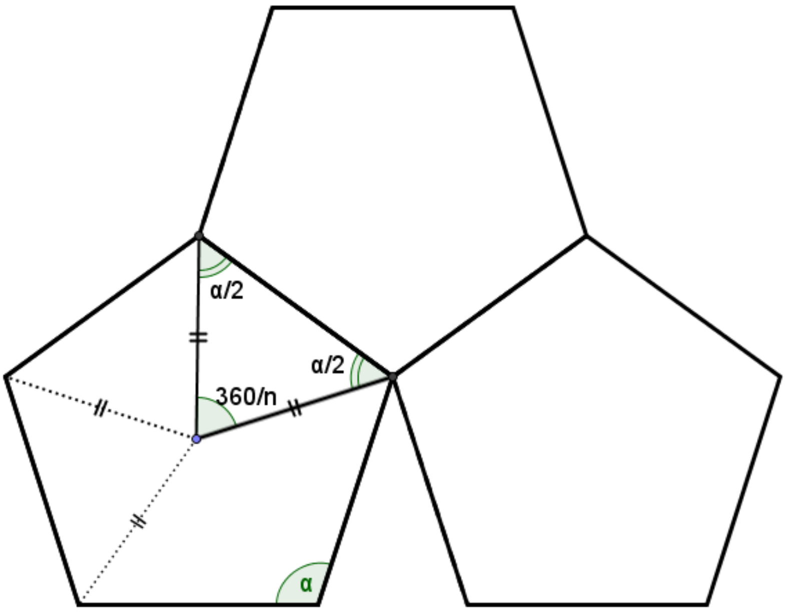
- ✓ Prepare a set of cards of two different colors: blue and yellow. Write the eight steps of the proof (sentences with gaps) on the blue cards and mathematical expressions on the yellow cards.
- ✓ Learners have to complete the sentences written in table 1, using a mathematical expression from table 2.
- ✓ Afterwards, they have to put the full sentences in the correct order.
- ✓ Prepare a correction where connection words and expressions used in mathematical proofs will be written in bold. (Make enough copies for each learner)

Teaching Sequence about Tessellation **<u>Activity 4</u>: A proof**	
<u>Doc.1</u>	**Why do the only regular tilings consist of equilateral triangles, squares and hexagons?**

It is easy to check that equilateral triangles, squares and regular hexagons can produce regular tilings.
We want to show that only those three regular polygons (that is to say regular polygons with 3, 4 or 6 sides) can make up regular tilings.

a) You can find below the beginning of the proof, and a diagram which illustrates the problem and the notations used.

"As shown in the diagram, we consider a regular polygon with n sides $(n \geq 3)$.
Let α denote the interior vertex angle of the regular polygon.
Then we divide our polygon into n isosceles congruent triangles."



b) The eight steps of the proof are written in table 1.
Organise the cards, in order to obtain the proof, and at the same time, complete the sentences from table 1 with the appropriate mathematical expression from table 2.

Teaching Sequence about Tessellation **<u>Activity 4</u>: A proof**	
<u>Doc.2</u>	**The eight steps of the proof *(table 1)* and the mathematical expressions *(table 2)*.**

***Table 1*:**

Yet the only divisors of 4 are …
However, at each vertex, k congruent polygons (k positive integer), with n sides must meet with no gaps, nor overlaps, so we have $k\times\alpha=$ …
$2+\dfrac{4}{n-2}$ is a whole number if and only if $n-2$ is a divisor of …
Since the sum of the three angles in a triangle has to be $180°$, we can deduce the following equality: … and therefore $\alpha=$…
Consequently the angles of those congruent isosceles triangles are respectively equal to …
Therefore that gives us only three values for $n-2$, and then three possible values for n : …
Assuming that $k=$…, we are looking for regular polygons for which the ratio $\dfrac{360}{\alpha}$ is a whole number.
However we have: $\dfrac{360}{\alpha}=\dfrac{360}{180-\dfrac{360}{n}}=\dfrac{360n}{180n-360}=\dfrac{2n}{n-2}=2+\dfrac{4}{n-2}$

Table 2:

$180-\frac{360}{n}$	$\frac{\alpha}{2}$, $\frac{\alpha}{2}$ and $\frac{360}{n}$
$\frac{\alpha}{2}+\frac{\alpha}{2}+\frac{360}{n}=180$	1, 2 and 4
4	360
$\frac{360}{\alpha}$	3, 4 and 6

Teaching Sequence about Tessellation **<u>Activity 4</u>: A proof** **Correction**	
<u>Doc.3</u>	**Why do only regular tilings consist of equilateral triangles, squares and hexagons?**

- As shown in the diagram, we consider a regular polygon with n sides $(n \geq 3)$.
  Let α **denote** the interior vertex angle of the regular polygon. Then we divide our polygon into n isosceles congruent triangles.

- **Consequently** the angles of those congruent isosceles triangles are respectively equal to ....

- **Since** the sum of the three angles in a triangle has to be $180°$, **we can deduce** the following equality: .... and therefore $\alpha =$ ....

- **However**, at each vertex, k congruent polygons (k positive integer), with n sides must meet with no gaps, nor overlaps, **so** we have $k \times \alpha =$ ....

- **Assuming that** $k =$ ...., we are looking for regular polygons for which the ratio $\frac{360}{\alpha}$ is a whole number.

- **However** we have: $\frac{360}{\alpha} = \frac{360}{180 - \frac{360}{n}} = \frac{360n}{180n - 360} = \frac{2n}{n-2} = 2 + \frac{4}{n-2}$

- $2 + \frac{4}{n-2}$ is a whole number **if and only if** $n-2$ is a divisor of ....

- **Yet** the only divisors of 4 are ....

- **Therefore** that gives us only three values for $n-2$, and then three possible values for n: ....

- **We can conclude that** only regular tilings are made up with regular polygons with 3, 4 and 6 sides that is to say equilateral triangles, squares and hexagons.

Teaching Sequence about Tessellation

Appendix: Student's Version

Activity 1: Guessing the lesson

Activity 2: Focus on language

Activity 3: Writing frame

Activity 4: A proof